Entanglement entropies in conformal systems with boundaries

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Entanglement Entropy in Many Body Quantum Systems
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SCUOLA
NORMALE
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INO
ISTITUTO NAZIONALE
DI OTTICA

Work in collaboration with:

  

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Francisco C. Alcaraz (Instituto de Física de São Carlos, Universidade de São Paulo)

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LT, J. C. Xavier. F. C. Alcaraz, and G. Sierra, PRB **88**, 075112 (2013) (see also LT, arXiv:1309.4003)

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Why study entanglement in many body physics?

- Understanding correlations is hard: one needs non-local quantities in order to fully get what is happening in the system

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- As $n \to 1$: von Neumann entanglement entropy $S = -\mathrm{Tr}_A \left[\rho_A \ln \rho_A\right]$

Outline

Rényi entropies and CFT

- Seminal result [Holzhey *et al.* '94]: logarithmic violation of the area law: for a finite interval in an infinite 1D chain,

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$$S_n = \frac{c}{6\eta}\left(1 + \frac{1}{n}\right) \ln\left[\frac{\eta L}{\pi} \sin \frac{\pi l}{L}\right]$$

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- These results were checked numerically in a plenty of models, and constitute now the easiest way of extracting c from simulations

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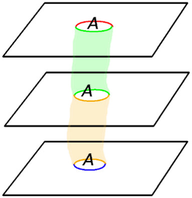

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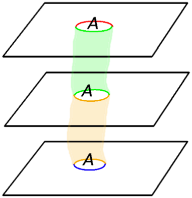

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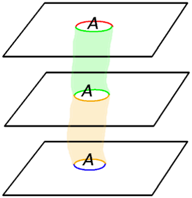

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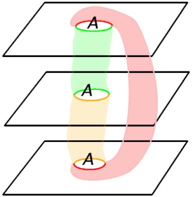

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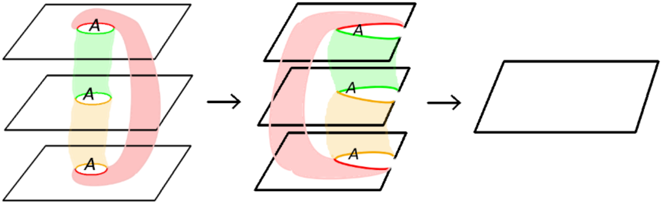

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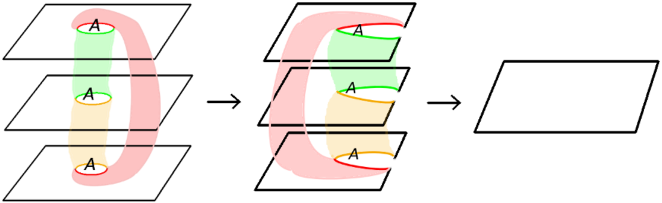

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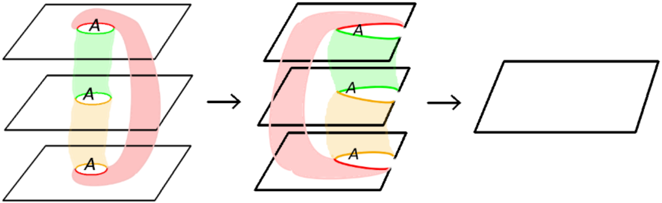

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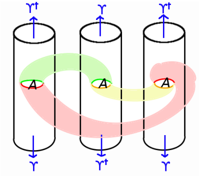

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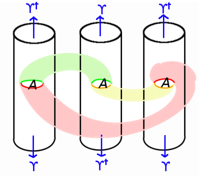

- The excess entropy wrt the GS, $F^{(n)}_{\Upsilon}(x) \equiv \mathrm{Tr}\rho^n_{A,\Upsilon}/\mathrm{Tr}\rho^n_{A,\mathbb{I}}$, takes the form

$$F^{(n)}_{\Upsilon}(x) = \lim_{w_k\to -i\infty} \frac{\left\langle \prod_{k=0}^{n-1} \Upsilon(w_k, \bar{w}_k)\,\Upsilon^{\dagger}(-w_k, -\bar{w}_k)\right\rangle_{\mathcal{R}_n}}{\langle \Upsilon(w_0, \bar{w}_0)\,\Upsilon^{\dagger}(-w_0, -\bar{w}_0)\rangle^n_{\mathcal{R}_1}}$$

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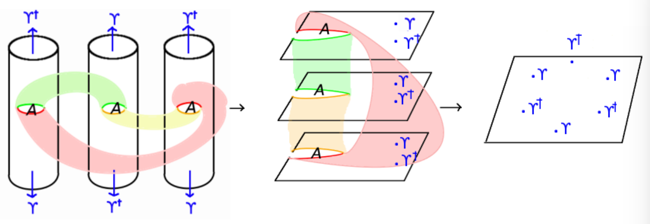

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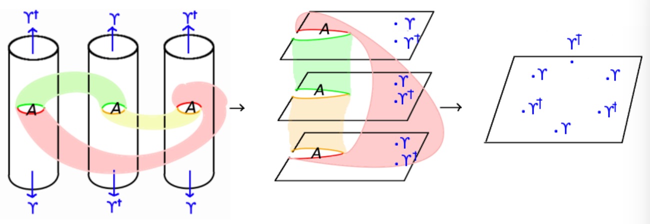

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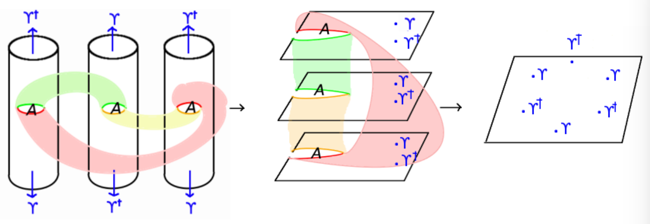

- The operators are placed at $z^{\pm}_{n,k} = \exp\left[\frac{i\pi}{n}(\pm x + 2k)\right]$, with $x \equiv \frac{l}{L}$
- The excess entropy is now

$$F^{(n)}_{\Upsilon}(x) = n^{-2n(h+\bar{h})} \frac{\left\langle \prod_{k=0}^{n-1} \Upsilon\left(z^{+}_{n,k}, \bar{z}^{+}_{n,k}\right) \Upsilon^{\dagger}\left(z^{-}_{n,k}, \bar{z}^{-}_{n,k}\right) \right\rangle}{\left\langle \Upsilon\left(z^{+}_{1,0}, \bar{z}^{+}_{1,0}\right) \Upsilon^{\dagger}\left(z^{-}_{1,0}, \bar{z}^{-}_{1,0}\right) \right\rangle^{n}}$$

where the correlators are on $\mathbb{C}$

Question: what happens when one considers *open* systems?

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- In general, the partition function of a rational model is given by $Z = \sum_{h,\bar{h}} \mathcal{M}_{h,\bar{h}} \times \chi_h \times \chi_{\bar{h}}$, $\mathcal{M}_{h,\bar{h}}$: multiplicity of the Verma module $h \otimes \bar{h}$

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- Boundaries reduce the operator content of the theory: just one chirality survives, and

$$Z_{\alpha\beta} = \sum_h \mathcal{N}^h_{\alpha\beta} \chi_h$$

$\mathcal{N}$: fusion coefficients $\rightarrow$ correspondence between boundary conditions and primary fields

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- Consequence: the GS of a system with boundary conditions α, β is obtained from the GS with free BC (FBC) by applying on it a chiral primary operator Υ (see also H. Saleur's talk)

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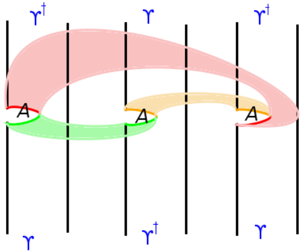

Entanglement entropies and boundary CFT (II)

- F can be simplified by means of the conformal transformations ($\mathcal{R}_n \to \mathbb{D}_n \to \mathbb{D}$)

$$w \to \frac{\sin\frac{\pi(w-l)}{2L}}{\sin\frac{\pi(w+l)}{2L}} \to z = \left[\frac{\sin\frac{\pi(w-l)}{2L}}{\sin\frac{\pi(w+l)}{2L}}\right]^{\frac{1}{n}}$$

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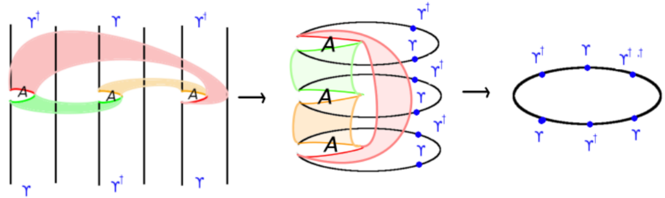

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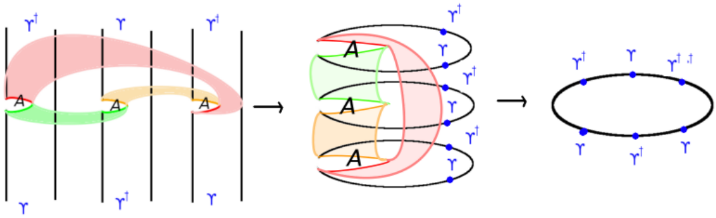

- Correlators on the disk become then correlators on $\mathbb{C}$:

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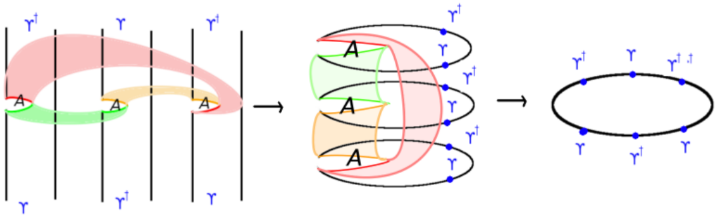

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- In principle, we have a recipe to compute corrections!

Critical Ising chain (I)

- Spin-1/2 Ising chain:

$$H = -\frac{1}{2}\left(\sum_{j=1}^{L-1}\sigma_j^x\sigma_{j+1}^x + h\sum_{j=1}^{L}\sigma_j^z\right)$$

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- Primary fields: $\mathbb{I}$ $(h = 0)$, σ $(h = \frac{1}{16})$, χ $(h = \frac{1}{2})$

Critical Ising chain (II)

- Conformal boundary conditions: $+$ (0), $-$ $(\frac{1}{2})$ and F $(\frac{1}{16})$, corresponding to fixing σ^x to $+1/2$, $-1/2$ or letting it free at the boundary [Cardy '86, Zhou *et al.* '06]

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- All the needed correlation functions for the $c = 1/2$ minimal CFT are known [Ardonne and Sierra '10, Berganza *et al.* '12, Essler *et al.* '13] (see also F. Essler's talk)

Critical Ising chain: numerics (I)

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- In all other cases: DMRG simulations (thanks to Fabio Ortolani for providing the code)

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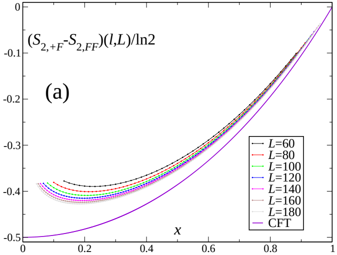
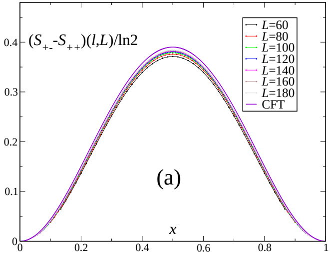
- $+$ and $-$BC can be implemented exactly with DMRG, by adding "ghost" sites at the edges

Critical Ising chain: numerics (I)

- For the FF case: free-fermions techniques [Vidal *et al.* '03, Peschel '03]
- In all other cases: DMRG simulations (thanks to Fabio Ortolani for providing the code)
- $+$ and $-$BC can be implemented exactly with DMRG, by adding "ghost" sites at the edges
- CFT: $F_\sigma^{(2)} = \cos\frac{\pi x}{4}$, $\lim_{n\to 1}\frac{1}{1-n}\ln F_\chi^{(n)}(x) = \ln|2\sin(\pi x)| + \psi\left(\frac{1}{2\sin(\pi x)}\right) + \sin(\pi x)$





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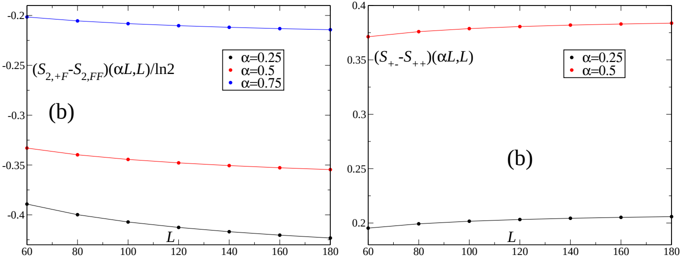

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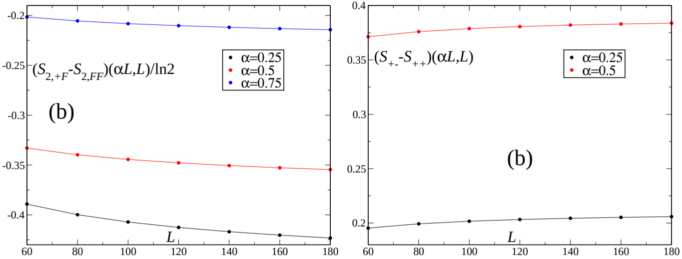

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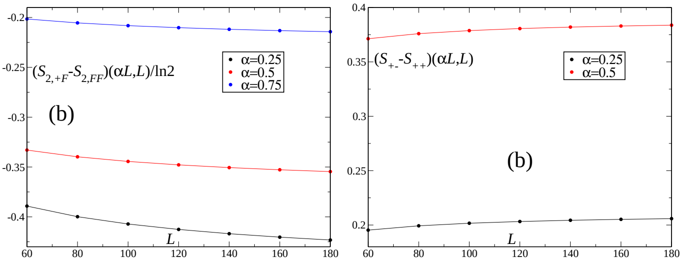

- Only thing we cannot predict: constant (and completely known) boundary entropies (BE's) [Affleck and Ludwig '91]: we add them to the CFT predictions
- In any considered case, excellent agreement between CFT and numerics

XX chain (I)

- BC cannot be implemented “exactly” with DMRG, and one has to consider the modified Hamiltonian [Bilstein '00]:

$$H = -\sum_{j=1}^{L-1}\left(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y\right) +$$
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- Lattice realization of conformal BC: D is realized by setting all boundary couplings to 0, N (on the left edge) by, e.g., α_-, $\alpha_+ \neq 0$

The spin-$\frac{1}{2}$ XX chain (II)

- Partition functions:

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\begin{aligned}
Z_{DD}(q) =& K_0(q) + K_2(q) = q^{-\frac{1}{24}}\left[1 + 2q^{\frac{1}{2}} + q + O(q^2)\right] \\
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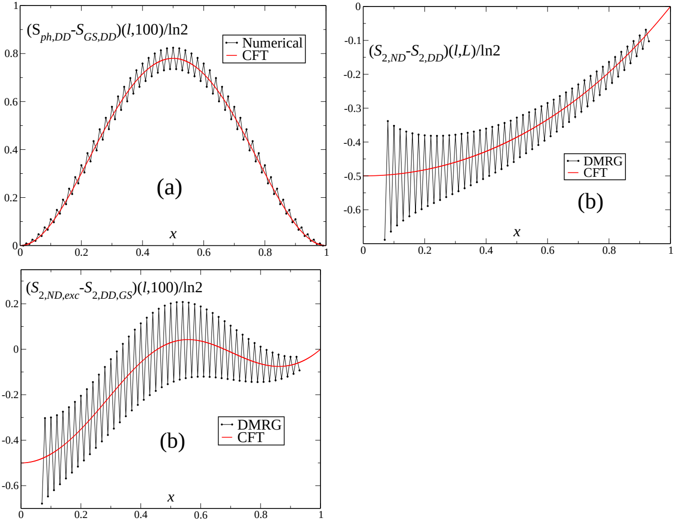

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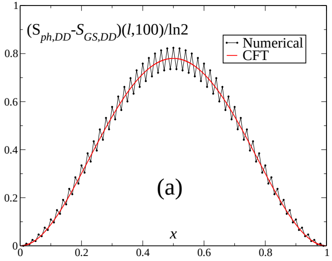

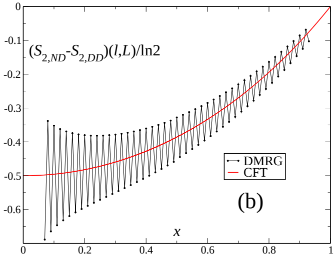

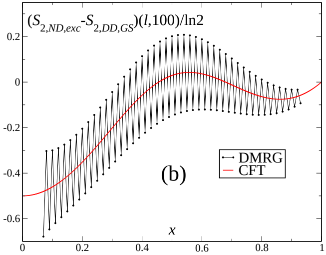

Everything can be understood by means of σ and $i\partial\phi$ correlators

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- Message: boundaries reduce the operator content of CFT $\rightarrow$ open models are advantageous playgrounds for the testing of CFT predictions
- Example of possible open issue: deriving the form of the corrections for a generic descendant state in CFT (work in progress!) and of the oscillating corrections

Thank you for the attention!