DYNAMICS OF ENTANGLEMENT CROSSING A QUANTUM PHASE TRANSITION

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OUTLINE

- Dynamics across a Quantum Phase Transition
- Entanglement in Many Body Systems
- Dynamics of Entanglement Entropy
- A case study: the Ising model in transverse magnetic field

  (PRB 89, 104303, 2014)

Dynamics of an isolated system

Theoretical challenges

What are the laws that rule the behaviour of a quantum system that is put in a non-equilibrium state?

- Dynamics and out of equilibrium properties of classical systems have been extensively studied
- For a (closed) quantum systems this is still an open problem

towards equilibrium: thermalization?

(generalized) Gibbs ensemble?

conserved quantities: integrable vs. non-integrable?

trapping in metastable (topological) states?

A. Polkovnikov, K. Sengupta, A. Silva, M. Vengalattore, Rev. Mod. Phys. 83, 863 (2011)
M.A. Cazalilla, M. Rigol (ed.), Focus on dynamics … , New Journal of Physics 12 (2010)

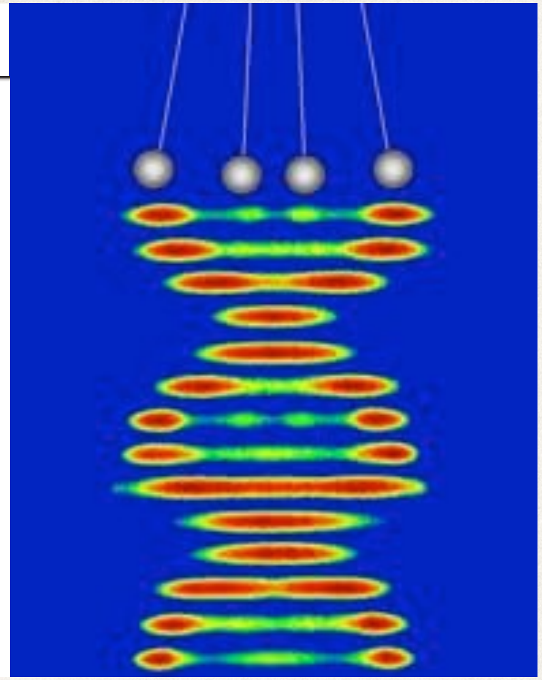
Experiments (with cold atoms)

Nature 440, 900 (2006)

A quantum Newton's cradle

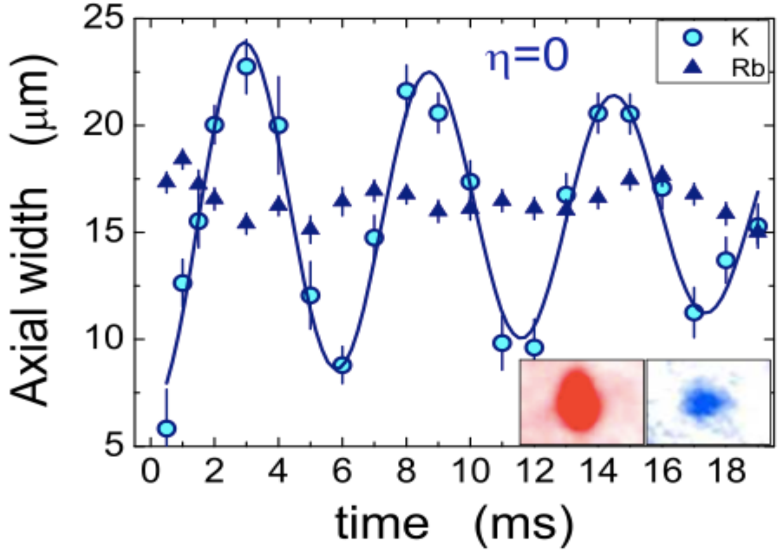
Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]

..... Here we report the preparation of out-of-equilibrium arrays of trapped one-dimensional (1D) Bose gases, each containing from 40 to 250 ^{87}Rb atoms, which do not noticeably equilibrate even after thousands of collisions. Our results are probably explainable by the well-known fact that a homogeneous 1D Bose gas with point-like collisional interactions is integrable.



Quantum dynamics of impurities in a one-dimensional Bose gas

J. Catani,[1,2] G. Lamporesi,[1,2] D. Naik,[1] M. Gring,[3] M. Inguscio,[1,2] F. Minardi,[1,2,*] A. Kantian,[4] and T. Giamarchi[4]



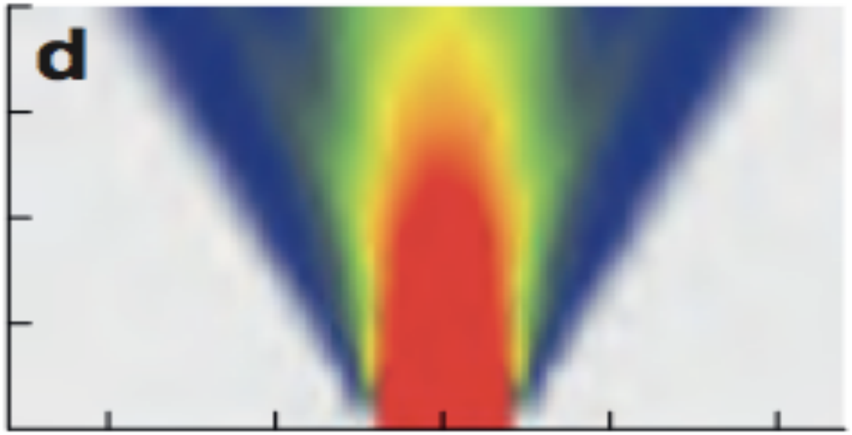
Using a species-selective dipole potential, we create initially localized impurities and investigate their interactions with a majority species of bosonic atoms in a one-dimensional configuration during expansion.

Phys. Rev. A 85, 023623 (2012)

Experiments (with cold atoms)

Expansion Dynamics of Interacting Bosons in Homogeneous Lattices in One and Two Dimensions

J. P. Ronzheimer,[1,2] M. Schreiber,[1,2] S. Braun,[1,2] S. S. Hodgman,[1,2] S. Langer,[3,4] I. P. McCulloch,[5] F. Heidrich-Meisner,[3,6] I. Bloch,[1,2] and U. Schneider[1,2]



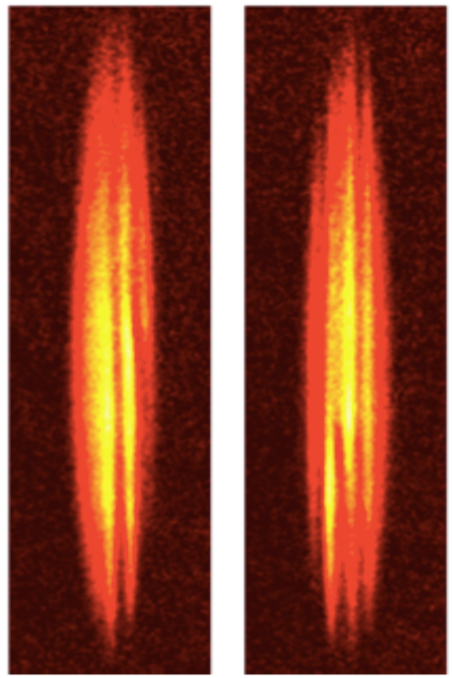
We experimentally and numerically investigate the expansion of initially localized ultracold bosons in homogeneous one- and two-dimensional optical lattices. We find that both dimensionality and interaction strength crucially influence these nonequilibrium dynamics.

Phys. Rev. Lett. 110, 205301 (2013)

Spontaneous creation of Kibble–Zurek solitons in a Bose–Einstein condensate

Giacomo Lamporesi, Simone Donadello, Simone Serafini, Franco Dalfovo and Gabriele Ferrari*

When a system crosses a second-order phase transition on a finite timescale, spontaneous sym- metry breaking can cause the development of domains with independent order parameters, which then grow and approach each other creating boundary defects. This is known as Kibble-Zurek mechanism. … Here we report on the spontaneous creation of solitons in Bose-Einstein condensates via the Kibble-Zurek mechanism. …

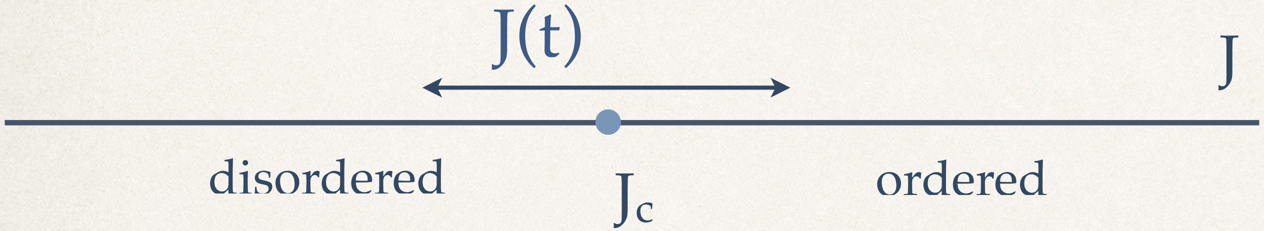


Nature Physics, 656, Vol. 9, 2013

Dynamics across a QPT

Theoretical set-up

universal behavior of dynamical physical quantities
(observables, correlation functions, ...) while crossing a QPT



$$H = H(J(t))$$

adiabatic change — finite v ramping — sudden quench

Formation of ordered domains and defects
while crossing a PT on a finite time-scale

originally introduced in cosmology

initially for (classical) PT in temperature

generalized to QPT

Rate of defect formation

$$n = (\xi^*)^{-d} \sim v^{d\nu/(z\nu+1)}$$

QPT's and Entanglement

- ground states of many body systems are highly entangled
- entanglement measures are able to detect phase transitions
- entanglement has a universal behaviour close to a QPT

Bipartite entanglement

$$|\psi_{GS}\rangle \Rightarrow \quad \rho = |\psi_{GS}\rangle\langle\psi_{GS}|$$

$$\rho_A \equiv Tr_B[\rho]$$

Von Neumann Entropy & Entanglement Spectrum

Subsystem A represents a quantum system in the B environment
& the reduced density matrix looks as that of a thermal state

$$\text{entanglement spectrum} \quad \omega_\alpha \quad \text{of} \quad \rho_{\mathrm{A}} \;, \quad \text{s.t.} \sum_\alpha \omega_\alpha = 1$$

von Neumman entropy $\quad S_{\mathrm{VN}} = -\sum_\alpha \omega_\alpha \log \omega_\alpha$

Schmidt gap $\quad \Delta_{\mathrm{S}} = \omega_1 - \omega_2$

The Model

Ising chain in transverse field

$$H = -\frac{1}{2}\sum_{j=1}^{L}\left[\sigma_j^z\sigma_{j+1}^z + h(t)\sigma_j^x\right]$$

QPT at $h_c = 1$

from a paramagnetic (h>>1) phase $|\rightarrow \cdots \rightarrow\rangle_{A\cup B}$

to a ferromagnetic (h<<1) phase $|\uparrow \cdots \uparrow\rangle_{A\cup B}$

Finite velocity ramping of the field

$$h(t) = h_i + \mathrm{sgn}(h_f - h_i)\frac{t}{\tau}$$

$$\frac{1}{v} \equiv \tau = 0.1 \to 500 \qquad h_i = 1.4 \to h_f = 0.4$$

Analytical calculations

Mapping into a fermionic Hamiltonian, via a Jordan-Wigner transformation

$$H = -\frac{1}{2}\sum_{j=1}^{L}\left[\left(c^{\dagger}_{j+1}c_j + c_{j+1}c_j + \mathrm{h.c.}\right) - 2hc^{\dagger}_{j}c_j\right] - \frac{Lh}{2}$$

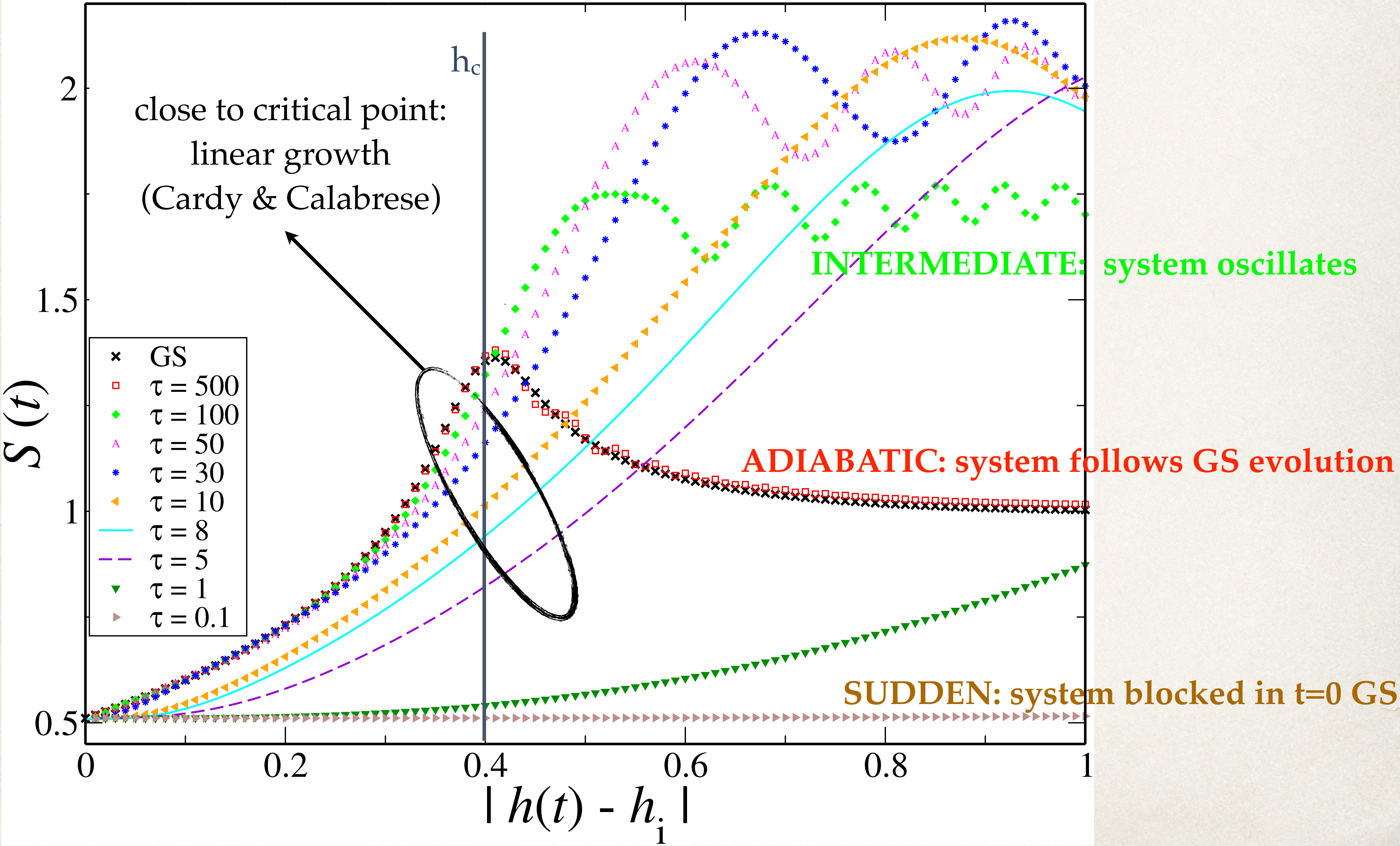
Diagonalizing via Fourier transform

and Bogolyubov transformation

$$H = \sum_{m=0}^{L-1} E_m\left(b^{\dagger}_m b_m - \frac{1}{2}\right)$$

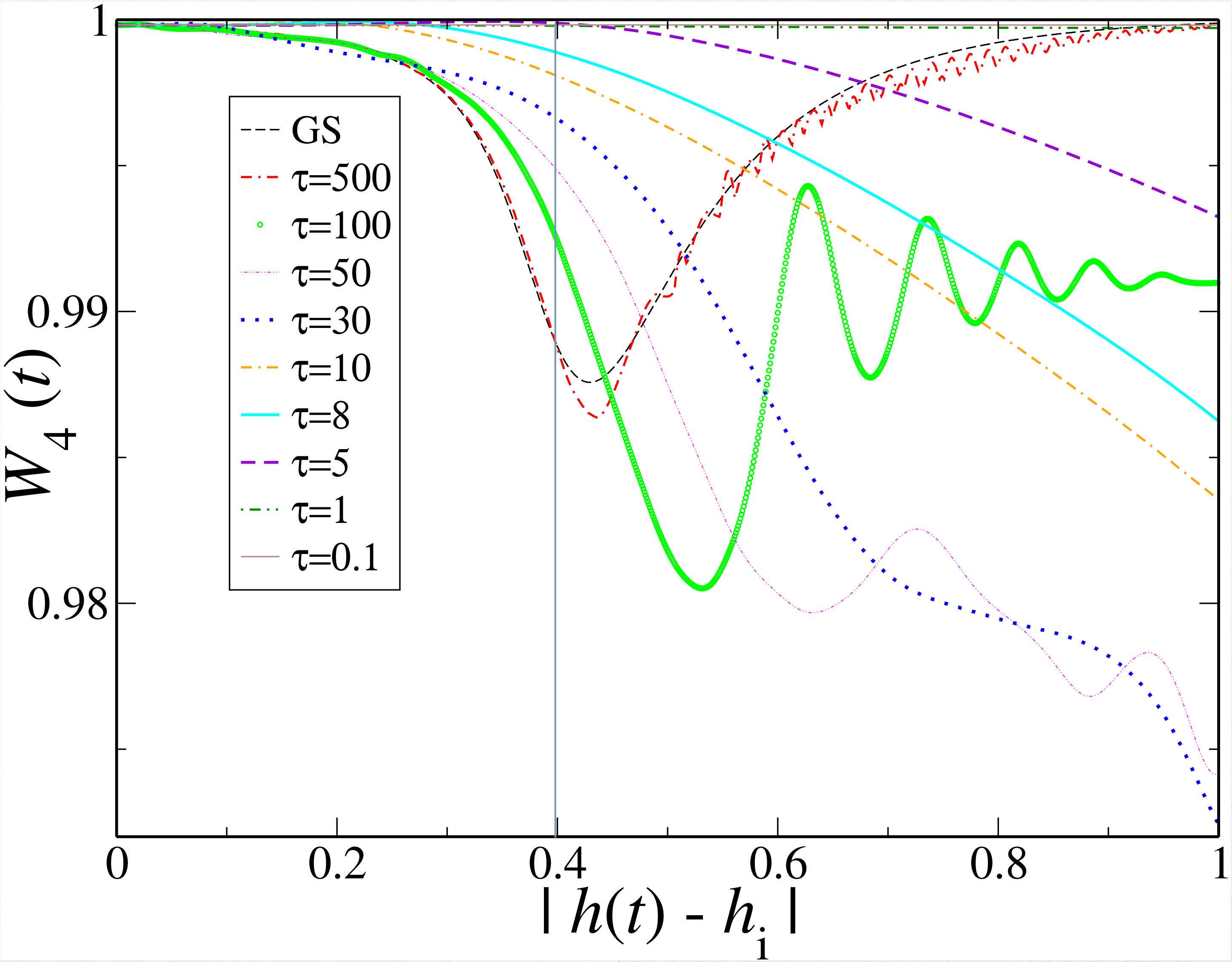
$$E_m = \sqrt{(h - \cos p_m)^2 + (\sin p_m)^2}$$

RESULTS: the von-Neumann entropy



four state dynamics where only first four eigenvalues are relevant

sum of first 4 eigenvalues



WHY: hints from perturbation theory

for h $\gg 1$ $$|0\rangle_{A\cup B} \equiv |\rightarrow \cdots \rightarrow\rangle_{A\cup B}$$

for h > 1 $$|GS\rangle_{A\cup B} = \mathcal{N}\left[|0\rangle_{A\cup B} + \frac{1}{4h}\sum |\rightarrow \cdots \uparrow\uparrow \cdots \rightarrow\rangle_{A\cup B}\right]$$

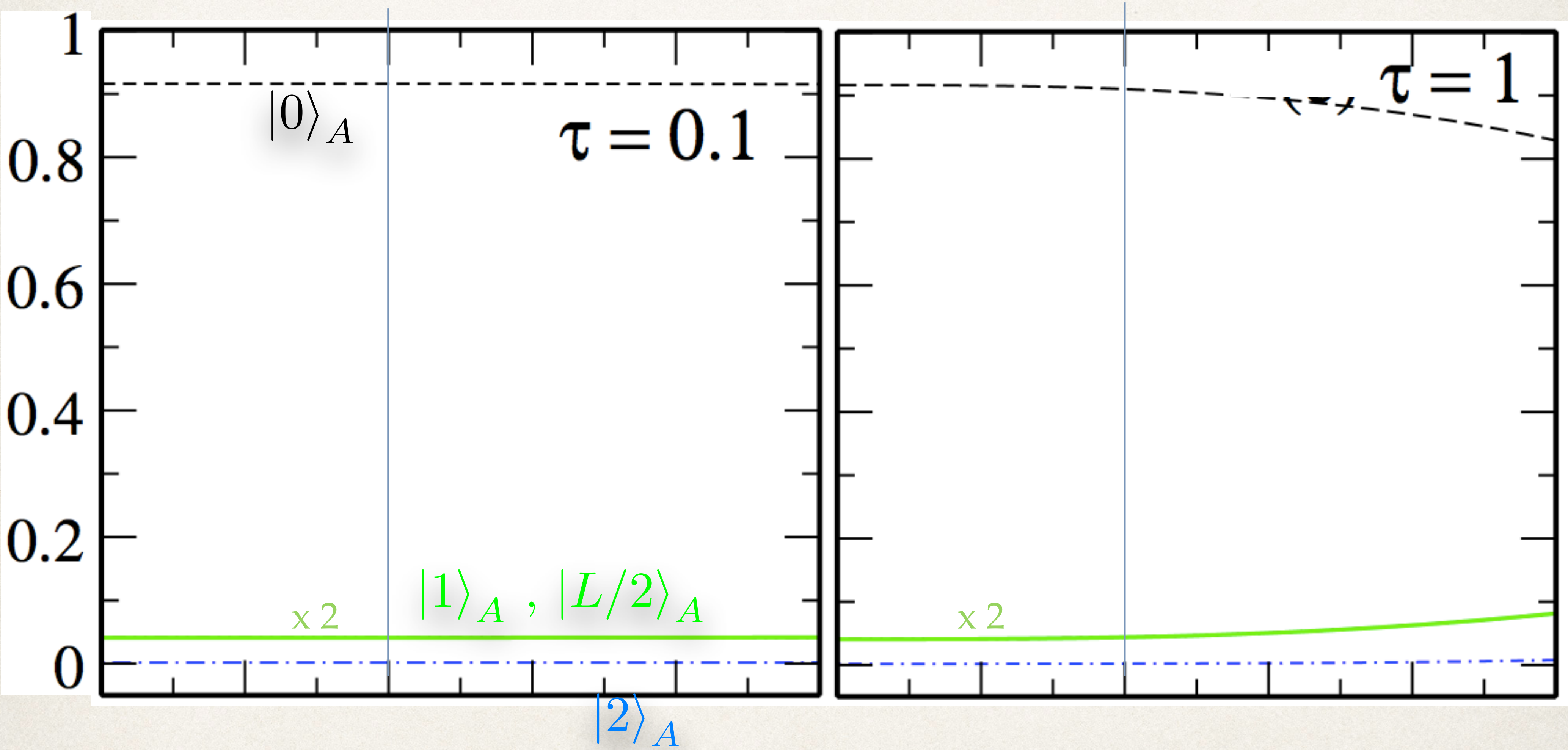
Tracing out B, there are 4 kinds of states contributing to the density matrix of A (half chain, PBC's):

$$|0\rangle_A \equiv |\rightarrow \cdots \rightarrow\rangle_A \qquad |1\rangle_A \equiv |\uparrow\rightarrow \cdots \rightarrow\rangle_A$$

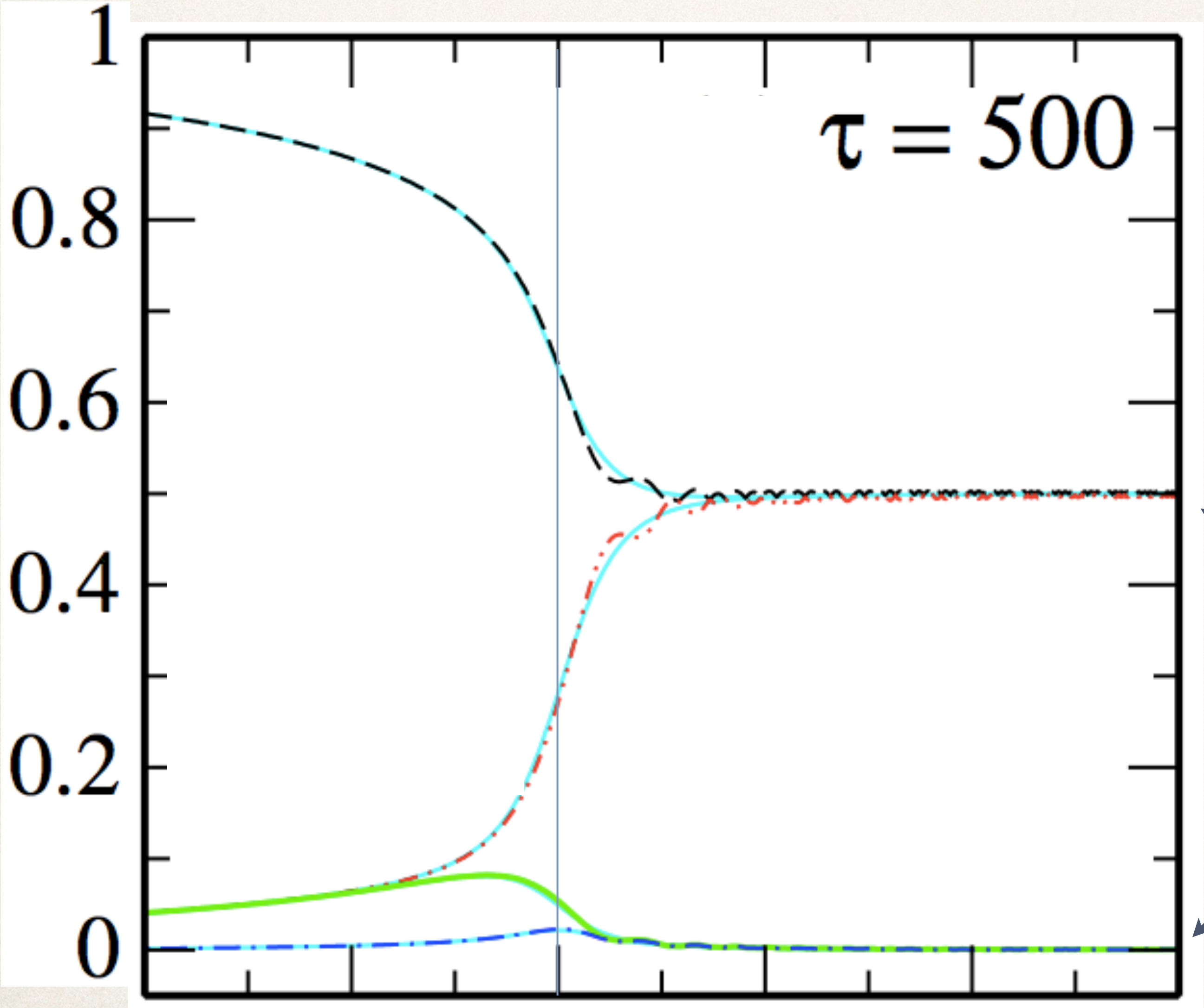
$$|2\rangle_A \equiv \sum |\rightarrow \cdots \uparrow\uparrow \cdots \rightarrow\rangle_A \qquad |L/2\rangle_A \equiv |\rightarrow \cdots \rightarrow\uparrow\rangle_A$$

(degenerate)

Entanglement spectrum in the QUENCH regime

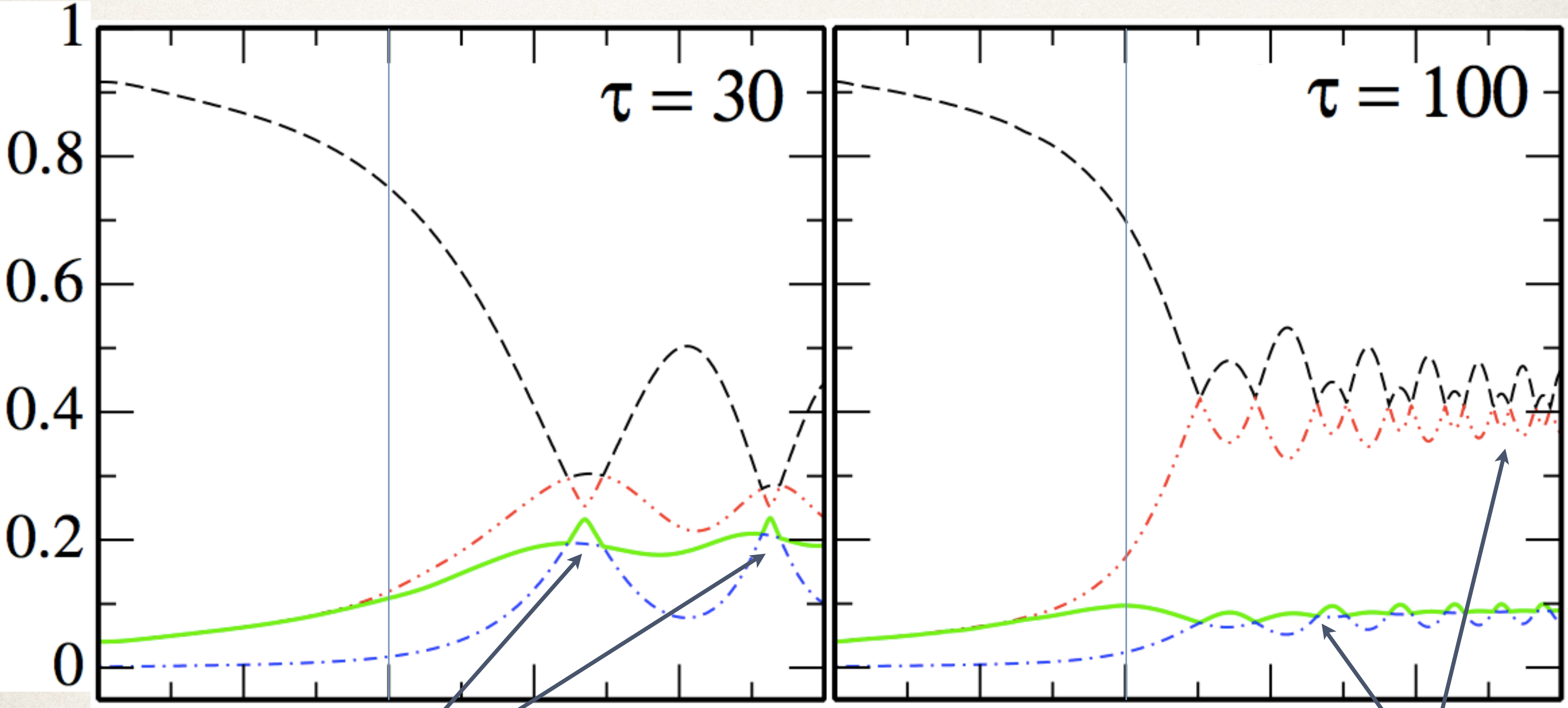


Entanglement spectrum in the ADIABATIC regime



Z_2 symmetry of Ising model at h=0

Entanglement spectrum in the INTERMEDIATE regime



Z_2 symmetry

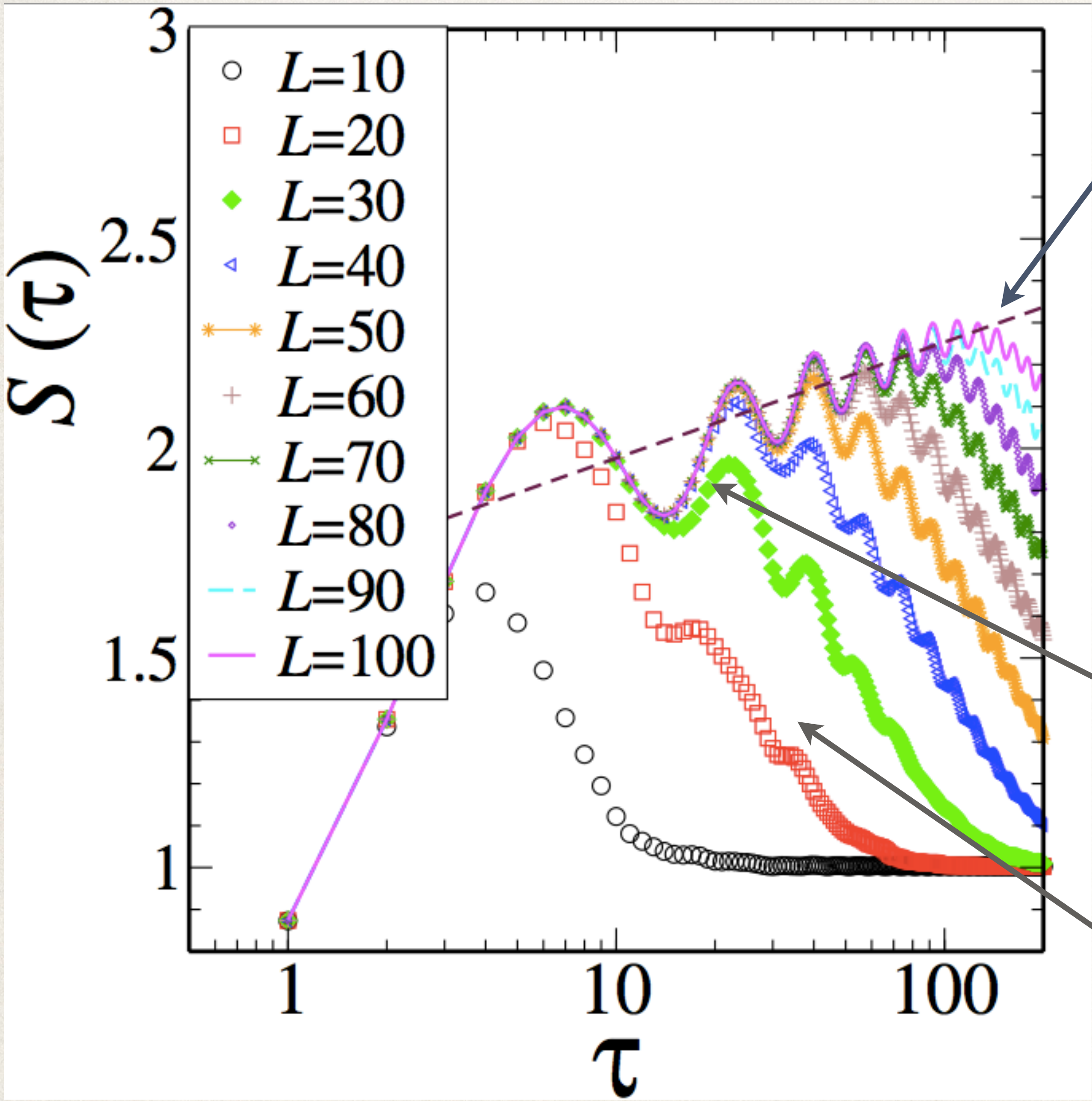
level crossing => maximum entropy

Universality: Kibble-Zurek physics

For a finite velocity, the evolution can be divided into three parts:
- a first *adiabatic* one, where the wave function of the system coincides with the ground state of H(t);
- a second *impulsive*, where the wave function of the system is practically frozen, due to the large relaxation time close to the critical point;
- a third *adiabatic* one, as the system is driven away from the critical point.

Entanglement entropy must have a universal behaviour:

$$S = \frac{c\nu}{6(1+z\nu)} \log \tau + const.$$



Agreement with theoretical predictions:

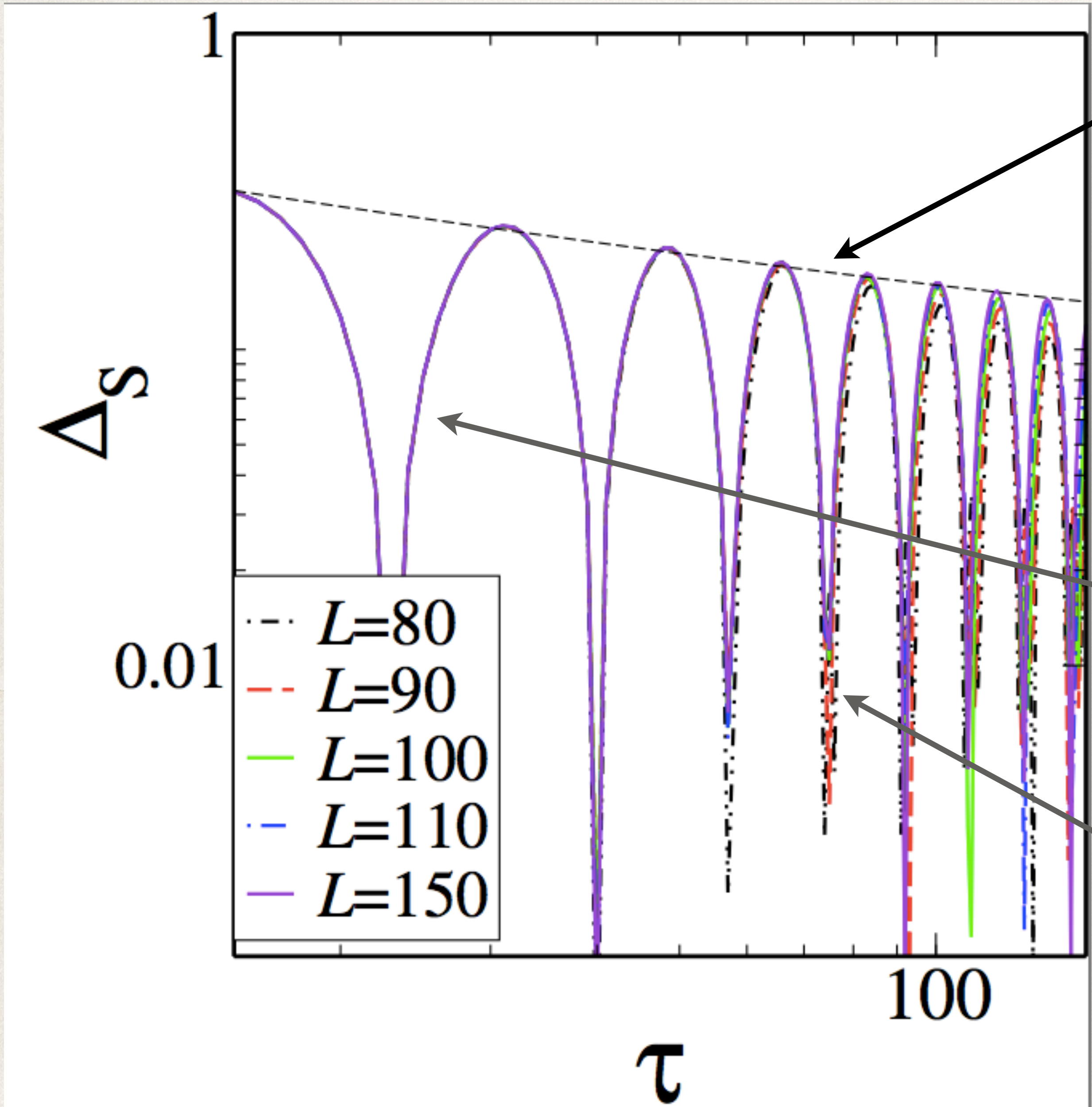
$$S = \frac{c\nu}{6(1+z\nu)} \log \tau + const.$$

$$(\ \nu = z = 1 \quad and \quad c = 1/2\ )$$

Critical linear behavior is superimposed to oscillatory one

Finite Size corrections are important

Universality: Schmidt gap



Agreement with theoretical predictions:

$$\Delta_S \equiv \omega_1 - \omega_2 \approx \tau^{-\frac{z\nu}{1+z\nu}}$$

(G. De Chiara et al., PRL 109, 237208 (2012))

Oscillatory behavior

(F. Pollmann et al., PR E81, 0201101 8 (2010))

Non-analiticity emerging from level crossing

(G. Torlai et al., arXiv:1311.5509)

OUTLOOKS

- Studied the dynamics across a Quantum Phase Transition via entanglement entropy & entanglement spectrum

A case study: the Ising model in magnetic field

- Analytical + t-DMRG (F.Ortolani, C. Degli Esposti Boschi):
    - fully interacting (still integrable models): e.g. XXZ, XYZ chain
    - higher symmetry models: e.g. SU(2) or SU(3) chains
    - disordered systems (with or without breaking of integrability)
    - spin chains & Bose-Hubbard models with bound states